Area of a Triangle

There are several ways to find the area of a triangle. The most basic way, of course, is the **<u>formula we learned in grade school using the base and the height</u>**. We can think of every triangle as being **half of a rectangle**. Therefore, we use the formula for the area of a rectangle, but change the names of the segments and take half of the result.

Rectangle: A = lw (length times width)

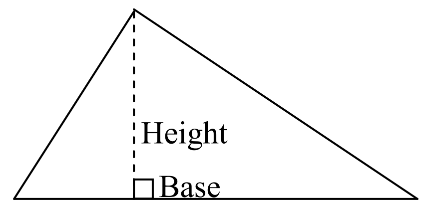
Triangle: A = ½ bh (base times height)

Example: Base = 12, Height = 7

A = ½ (12x7)

A = 6x7

A = 42 units squared (or square units)



Another method is **Heron's Formula** (sometimes called Hero's Formula). This method looks complicated, but it works **<u>when you know all three sides of a triangle</u>**.

Label the triangle sides a, b, and c.

Find the "Semi-perimeter" by adding a, b, and c, then dividing by 2. Call it "S."

Heron's formula states:

A = $\sqrt{S(S-a)(S-b)(S-c)}$

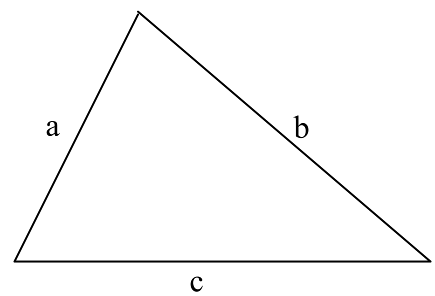
Example: a = 20, b = 21, c = 29

So the perimeter is 70 and S = 35.

Then A = $\sqrt{35(35-20)(35-21)(35-29)}$

A = $\sqrt{35x15x14x6}$

A = $\sqrt{44{,}100}$ = 210 Units^2



A third method works **<u>when you know any two sides and the measure of the angle between them</u>**. It requires a slight knowledge of Trigonometry, and either a "Trig" table or a calculator that can find the Sine of an angle.

Label the triangle sides a, b, and c, and the angle across from each side with the matching capital letter.

The formula is:

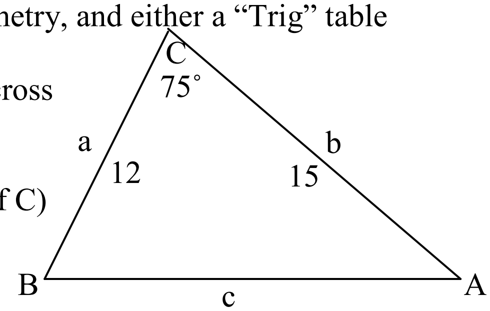
A = ½ abSinC (one-half a times b times the Sine of C)

Example: a = 12, b = 15, angle C = 75°

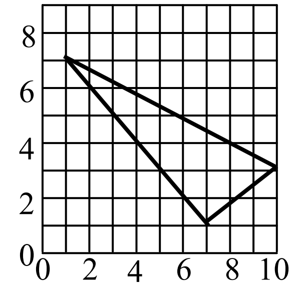
So A = ½ (12)(15)Sin(75)

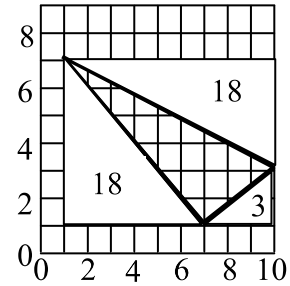
A = 90(.9659…)

A = 86.9 Units^2



If you know the coordinates of the vertices of a triangle, you have two methods available. The first is to create a rectangle which contains the triangle. For this example, our rectangle would need to be from (1, 1) to (10, 1) to (10, 7) to (1, 7). Find the area of this rectangle (6 blocks tall and 9 blocks wide for an area of 54 square units), then subtract the areas that are inside the rectangle but outside the triangle. The region from (1, 7) to (10, 7) to (10, 3) is a right triangle with legs 4 and 9, and thus, an area of 18. From (1, 7) to (1, 1) to (7, 1) has legs 6 and 6 and area 18. The small triangle from ((7, 1) to (10,1) to (10, 3) has legs 2 and 3 and area 3.



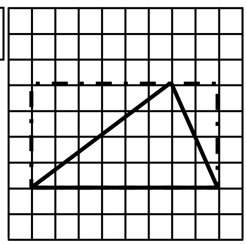


So 54 – 18 – 18 – 3 = 15 square units.

As seen in the diagram at right, the triangle's area is what remains when the rest of the rectangle is removed.

If **one side of the rectangle is also one side of the triangle,** then the triangle will be exactly one-half the area of the rectangle. Usually one side of the triangle is a vertical or horizontal segment. Observe the three examples below. Find the area of the rectangle and then divide it by two. In the third example, the triangle and rectangle have the same base and height, even though the rectangle does not totally contain the triangle.

1.



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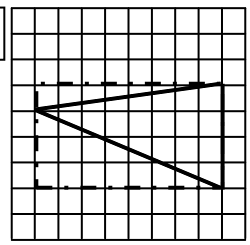

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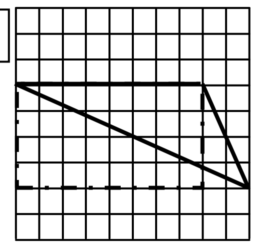

All three rectangles have an area of 32, so the triangles have an area of 16. In each case, the base and height are 8 and 4. We will use the next method to also find the area.

The second method requires less drawing. List the coordinates of the vertices in a vertical stack, going clockwise or counterclockwise around the triangle. List the beginning vertex again at the end to provide closure. Multiply diagonally down the stack to get one total, then diagonally back up the stack to get another. Subtract the two totals and divide by two.

So if we start our original triangle at (1, 7) and go around the triangle counterclockwise,
our stack is: 1, 7 and we multiply like this: 1, 7 and 1, 7 → 49
7, 1 → 1 then 7, 1 → 10
10, 3 → 21 10, 3 → 3
and 1, 7 → 70 1, 7 (62)
(92)

Subtract the two totals, then divide by two.
92 – 62 = 30 30 ÷ 2 = 15 square units.

For triangles 1, 2, and 3, the stacks look like this:

#1)			#2)			#3)	
1, 2	1, 2		1, 5	1, 5		0, 6	0, 6
9, 2	9, 2		9, 2	9, 2		10, 2	10, 2
7, 6	7, 6		9, 6	9, 6		8, 6	8, 6
1, 2	1, 2		1, 5	1, 5		0, 6	0, 6
70	38		101	69		108	76

70 – 38 = 32 32 ÷ 2 = 16 101 – 69 = 32 32 ÷ 2 = 16 108 – 76 = 32 32 ÷ 2 = 16

Two very handy formulas relating to triangles:
If you need to **find a missing part of a triangle**, the following formulas may help.

Law of Sines:

$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

Law of Cosines: $c^2 = a^2 + b^2 - ab\text{Cos}C$