

Estimating Square Roots

In order to successfully estimate square roots, you must first understand the “perfect squares.” See the list below. The numbers on the right side are perfect squares.

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144$
$13^2 = 169$
$14^2 = 196$
$15^2 = 225$

When you need to estimate a square root, locate where the number would fit in the perfect square list. The square root will fall between the integers on the left.

Example 1: Estimate the square root of 72.

We know 72 falls between 64 and 81 on the perfect square list. Its square root will fall between 8 and 9. Since 72 is about halfway between 64 and 81 ($72-64=8$ and $81-72=9$), the square root will be close to 8.5, maybe just a fuzz less than 8.5.

If we check our estimation, we see that we are pretty close. Our work looks like this:

$8^2 = 64$			
$?^2 = 72$	8	}	So 72 is $\frac{8}{17}$ of the way from 64 to 81.
$9^2 = 81$	17		$\frac{8}{17}$ in decimals is 0.470588...
			So our estimate is 8.47 and the square root is about 8.49.
Close Enough since both round off to 8.5!			

Example 2: Estimate the square root of 45.

Since 45 falls between 36 and 49, its square root will fall be greater than 6 but less than 7. It will be closer to 7 because 45 is closer to 49 ($45-36=9$ and $49-45=4$). From this I would predict the square root to be near 6.7. Again...

$6^2 = 36$			
$?^2 = 45$	9	}	So 45 is $\frac{9}{13}$ of the way from 36 to 49.
$7^2 = 49$	13		$\frac{9}{13}$ in decimals is 0.69230769...
			So our estimate is 6.69 and the square root of is about 6.71.
Close Enough since both round off to 6.7!			

Example 3: Estimate the square root of 147.

This number is between 144 and 169 on our list, so its square root will be more than 12 and less than 13. It is much closer to 144 than 169 ($147-144=3$ and $169-147=22$). I would estimate the square root to be between 12.1 and 12.2. Let's do the math...

$12^2 = 144$			
$?^2 = 147$	3	}	So 147 is $\frac{3}{25}$ of the way from 144 to 169.
$13^2 = 169$	25		$\frac{3}{25}$ in decimals is 0.12.
			And our estimate is 12.12, while the real square root is about 12.12. Wow!