Special Exponents

Negative Exponents

Sometimes a road sign gives you information.

Sometimes a road sign tells you what to do.

Sometimes a road sign does both!

Exponents can be the same way.

If an exponent is negative, it is like the "Wrong Way!" sign you see on interstate ramps. Basically, a negative exponent screams, "Wrong Place!" Since every rational number can be written in fraction form, if the negative exponent is in the numerator (or is on a whole number), we simply move the exponent and its base to the denominator. If it is in the denominator, we move it to the numerator.

Examples: x^{-2} becomes $\frac{1}{x^2}$ and $(3x)^{-4}$ becomes $\frac{1}{3^4x^4}$. Also, $2x^{-1}$ becomes $\frac{2}{x}$ and $3a^2b^{-2}c^{-1}$ becomes $\frac{3a^2}{b^2c}$

Examples where the negative exponent starts in the denominator:

$\frac{4a^2bc^{-3}}{x^2y^{-4}}$ becomes $\frac{4a^2by^4}{x^2c^3}$ In short, move the part with the negative exponent and leave the rest alone! The advantage is that fractions can be written on one line.

Fractional Exponents

A fractional exponent is just a different way to show square root (or any other root) notation. So if you see $x^{1/2}$ (x to the one-half power), it is the same as $\sqrt{x}$ (The square root of x).

If the exponent's numerator is not 1, then that part becomes the exponent of the number in the radical sign. So $x^{3/2}$ (x to the three-halves power) becomes $\sqrt{x^3}$, which simplifies to $x\sqrt{x}$. Since three halves is the same as one and one-half, we have one "whole" x and another x that is in a square root sign. In other words, if the fractional exponent will simplify, so will the radical term it represents. Think of the fraction as a tree; the top part is branches and the bottom is roots. This also works for numbers.

The notation $8^{2/3}$ (8 to the two-thirds power) means the <u>cube</u> root of 8 <u>squared</u>. So we can take the cube root of 8 (that's 2) and square it, or we can square the 8 (that would be 64) and then take the cube root. Either way, the answer is 4. Fortunately, both 8 and 64 have integral cube roots.

So if we do a number that is not as convenient, our answer doesn't come out as neatly. If we take $5^{3/4}$ (5 to the three-fourths power), it means "the fourth root of five to the third power (or 'cubed')." We have no convenient way to show fourth root, but it would be a radical sign with a small 4 in the notch. The simplest we could make this is the fourth root of 125. $\sqrt[4]{125}$

Study guide developed by Ladder Survivors, Inc.