Geometric Transformations

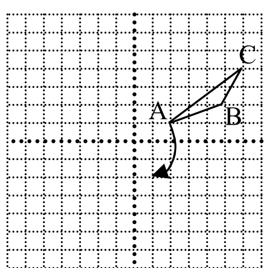
Geometric Transformations have three formats – Rotations, Reflections, and Translations. A third grade class once taught me their easy way to remember them:

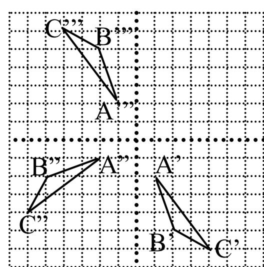
> Ro**t**a**t**ions have two t's, so we **t**urn **t**hem.
> Re**fl**ections has an fl, so we **fl**ip it.
> Tran**sl**ation has an sl, so we **sl**ide it.

I introduce transformations by using "pattie-paper," wax paper, or transparencies. Pattie paper can be purchased at most warehouse sales such as Sam's Club or Costco. It requires little prep time for use and is semi-transparent. Wax paper is easier to find and often cheaper, but requires more advance prep time to cut it to convenient sizes for student use. Transparencies cost more initially, but can be cleaned and reused if one wants to go to the trouble. The pattie-paper method helps the student to see the result and then assign new coordinate values. Eventually, many students become able to skip the manipulative stage and directly assign new values by using the pattern discovered.

Rotations: Every rotation has to have a "center of rotation." If none is specified, we assume the center of rotation is the origin (0, 0). Students place pattie-paper over the original grid and figure, and trace the axes and figure onto the pattie-paper. They then anchor the center of rotation with their pencil point and rotate the pattie-paper the required number of degrees, and in the correct direction. If the rotation is 90˚, 180˚, 270˚, or 360˚, they will find that the axes will again line up.

Example: Suppose we have a triangle with vertices A(2, 1), B(5, 2) and C(6, 4). If we rotate it 90˚, 180˚, and then 270˚ clockwise, the original and rotated images will look like this:



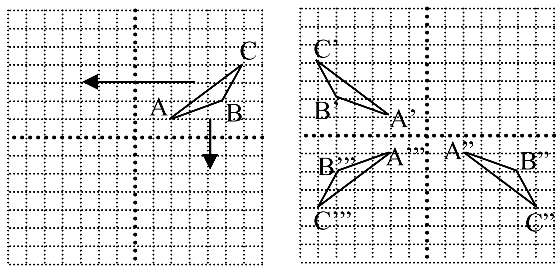


Pattern for 90˚ Rotation:
Notice that A(2, 1) became A'(1, -2), B(5, 2) became B'(2, -5) and C(6, 4) became C'(4, -6). So the pattern for any 90˚ clockwise rotation must be P(x, y) → P'(y, -x). If we had a point at (-4, 7), it would then rotate to (7, 4).

Similarly, a pattern for any rotation can be developed by observing an actual rotation. We say that rotation preserves size and shape, but not orientation, since the figure remains the same except for the direction it points.

Reflections: A reflection is a mirror image across a line of symmetry. If you look in a mirror or a puddle, the line of symmetry is the surface of the mirror or water. The reflected image will face in the opposite direction from the original. If you raise your right hand, your image will raise the left hand. Your left ear is the reflection's right ear. (This is why your school photos never look quite right to you, but look OK to your friends and family – your memory of yourself is of your mirror image, not the photo.) Using pattie-paper to do reflections, students place the paper over the original grid and image, and trace the image and the axis which will be used as the line of symmetry, marking the origin (0, 0) on their drawing. They then flip the drawing over and align the axis and origin with the original, so they are looking at the back side of their tracing.

Example: We will begin with our original triangle ABC. Our first reflection (A'B'C') will be across the y axis, A"B"C" will be across the x axis, and A'"B'"C'" will be a double reflection (it really doesn't matter which axis we do first – try it!).



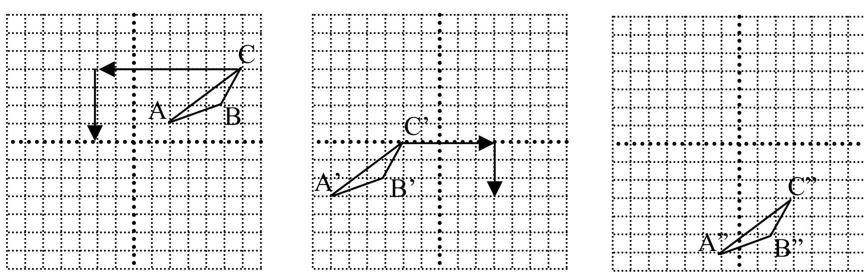
Reflection pattern(s):
When A(2, 1) is reflected across the y axis, it becomes A'(-2, 1). If we reflect A across the x axis, it becomes (2, -1). So the coordinate which matches the axis of symmetry stays the same, and the other coordinate changes signs.

Again, a student should use the manipulative (pattie-paper) to get the correct result, then analyze the result to formulate a pattern. Just like rotations, reflections preserve size and shape, but not orientation.

Translations: The word "translation" makes one think of foreign languages. If we move an object like an armchair from one country to another, its description would change and its location would change, but it would still be the same object as before. If you visualize this by using a world globe, sliding your finger from country to country to represent the move, you see that "translation" is a pretty good description.

Using pattie-paper, the student should trace the figure. Focus on one vertex. Move it according to the directions given, and the rest of the figure has to follow.

Example: Again using our original triangle ABC, we will show a couple of translations. We will translate the original triangle eight left and four down. We will then translate the resulting image (A'B'C') five right and three down.



Translation Pattern:
Notice that C began at (6, 4), moved 8 left and four down (to (-2, 0)), then 5 right and 3 down. We can do the geometry as an arithmetic problem: (6-8+5, 4-4-3) = (3, -3), which is the final coordinate of our point C. Likewise, point A starts at (2, 1), so (2-8+5, 1-4-3) = (-1, -6) and B(5, 2) becomes (5-8+5, 2-4-3) = (2, -5). With practice, we can do translations without using a graph at all. If we had a point at (12, 9) and did the same two translations on it, its final image would lie at (9, 2).

Translations preserve size, shape **and** orientation.

Sometimes Dilation is taught with the transformations, but pattie-paper does not help much with dilations, so we will save that for another study guide.