

Modulo Math

Take a minute to answer these questions:

In what mathematical system are the following statements true?

$12 + 12 = 12$

$10 + 4 = 2$

$3 - 6 = 9$

$4 + 24 = 4$

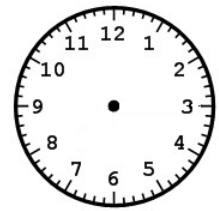
$11 + 6 = 5$

What if I told you that you use this system all the time?

You probably spend more time in this system than you spend using any other math.

The answer is “Modulo Math,” specifically Modulo 12, which is often called “Clock Math.”

Picture a clock face (Analog, not digital). Start at any number on the clock and add 12 (or any multiple of 12). You will come back to where you started. This explains why $12 + 12 = 12$ and $4 + 24 = 4$.



If you start at a number and add to it so the total is less than 12 (such as $5 + 4 = 9$), it looks just like “normal” math. But if the total is more than 12, we start over again at 1, and our answer is the new total minus 12. So using our examples, $10 + 4 = 14$, but since there is no 14 o’clock in the 12-hour system, we get $14 - 12 = 2$, so $10 + 4 = 2$ in Modulo 12. Also, $11 + 6 = 17$, but $17 - 12 = 5$, so $11 + 6 = 5$.

Now what about subtraction? Once again, if tracing the subtraction on a clock face does not take you past the 12, the problem appears “normal.” So $11 - 5 = 6$. However, if tracing the subtraction on a clock face takes you past the 12, you’ll get a problem that appears impossible, if you weren’t doing “clock math.” This explains why $3 - 6 = 9$. If it is now 3 o’clock, then six hours ago it was 9 o’clock.

When you’ve mastered the concepts of Modulo 12, you realize that 12 is not a magic number – we could use any number for the base of our modulo system. For example, in Modulo 5, these are true:

$4 + 4 = 3$

$1 - 2 = 4$

$2 \times 3 = 1$

$5 - 5 = 5$

$3 + 4 = 2$

$1 + 4 = 5$

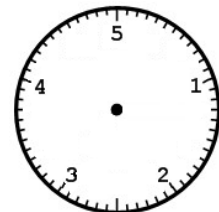
$2 + 4 = 1$

$3 + 4 = 2$

$2 \times 3 = 1$

$3 \times 3 = 4$

$4 \times 3 = 2$



In order to do Modulo math, just visualize a clock face with the modulo base as the highest number, or the number where 12 traditionally goes. If the computation takes you past the top number, start counting from there.

Now in plain English, here’s why it works: Perform whatever operation is required. Divide the result by the modulo base. If the remainder* is zero, your answer is the base value (In Modulo 12, $12 + 12 = 24$, $24/12 = 2$, remainder 0, so the answer is 12). If the remainder is any other number, that remainder is your answer. (In modulo 8, $6 + 5 = 11$, and $11/8 = 1$, remainder 3, so 3 is my answer.)

*In Modulo Math, the remainder is called a “residue.”