Number Bases

When "New Math" arrived, the emphasis on number bases increased. Until then the only base most of us knew was base ten – the decimal system (deci- is the Latin prefix for ten).

A number system can be based on **any** number, but some are more commonly used than others. We use base ten because we have ten fingers. If horses were the dominant intellectual life form on Earth, they would use base two.

The most frequently used number bases are:
Binary – base two
Octal – base eight
Decimal – base ten
Hexadecimal – base sixteen

In all base systems, the pattern is the same. To the left of the point (in base ten we call it a "decimal point," but in base two it would be a "binary point," and so on…) is the place value "ones." Then we have the base number, then the base number squared, the base number cubed, and keep going as far as the number extends. To the right of the point, we have one over the base, followed by one over the base squared, then one over the base cubed, and so forth. Graphically, the pattern would look like this for a base value of "x."

$$\ldots\ x^5\ \ x^4\ \ x^3\ \ x^2\ \ x^1\ \ x^0\ \ x^{-1}\ \ x^{-2}\ \ x^{-3}\ \ x^{-4}\ldots$$

So for base 10, the number 765,203.9418 would be seven hundred-thousands (10^5), six ten-thousands (10^4), five thousands (10^3), two hundreds (10^2), no tens (10^1), 3 ones (10^0 – **anything** to the zero power is **one**), and* nine tenths (10^{-1}), four hundredths (10^{-2}), one thousandth (10^{-3}), eight ten-thousandths (10^{-4}). We can show it like this:

7	6	5,	2	0	3.	9	4	1	8
10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
					one	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10,000}$

Negative exponents are just another way to do fractions.

In every base, the largest single digit possible is the digit that is one less than the base. So in base 10, the largest digit is 9. In base two, it is 1, and in base 16, we have to make up a digit to represent 15, so we use F. To count from one to 15 in base sixteen we would use 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F.

In every base, the pattern remains the same, even though the numbers change.

In Base Two (Binary), the place values would be:

2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
32	16	8	4	2	one	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

Therefore 100110.0101 would be $38\frac{5}{16}$.
$(1x32 + 0x16 + 0x8 + 1x4 + 1x2 + 0x1 + 0x\frac{1}{2} + 1x\frac{1}{4} + 0x\frac{1}{8} + 1x\frac{1}{16})$

That's $32 + 0 + 0 + 4 + 2 + 0 + 0 + \frac{1}{4} + 0 + \frac{1}{16} = 38\frac{5}{16}$

See our study guides for base two, base eight, and base sixteen.