

Using the Sieve of Eratosthenes

The definition of Prime Numbers contains two provisions. While everyone knows #2, sometimes #1 is overlooked.

1. All prime numbers are natural numbers **greater than one (1)**.
2. Every prime number has exactly two factors – one (1) and the number itself.

The Sieve of Eratosthenes is a method of locating Prime Numbers by filtering out all of the numbers which are **not** prime. You probably have a sieve in your home. However, you might call it a screen or a sifter. These tools allow fine material to pass through while trapping coarse material.



To use the Sieve of Eratosthenes, list the counting numbers as far as you wish. The sheet accompanying this study guide goes to 150. Your steps are as follows:

1. Since 1 is not a prime number, mark it out.
2. The number **2** meets both conditions, so circle it. Now look at all the other even numbers. Since 2 is a factor of all these other even numbers, they will not meet condition #2. So mark them out, starting with 4. This is easy to do on our chart since they are in nice straight columns. **So 2 is the only even prime number.**
3. Move on to number **3**. It meets both conditions, so circle it. Now go through and mark out all of the remaining multiples of 3. Since 6 is already marked out, 9 is the first new 'mark-out.' You'll probably notice that there is a pattern to the multiples of three.
4. Move on to **5**. It meets both conditions, so circle it. Now mark out all the other multiples of 5 which haven't already been crossed out. You'll notice that 25 is the first new mark-out. See a pattern?
5. Move on to **7**. Circle it and mark out the other multiples of 7. Now 14 was already marked and so was 21, 28, etc. What is your first new mark-out?
6. Some people are thinking that your odd numbers are prime, but you see that 9 is marked out (when we did '3s'). So move to **11**. Circle it and mark out the other multiples of **11**. See a pattern? What is your first new mark-out?
7. Continue this pattern until you cannot mark out any new numbers. At this point any unmarked numbers will be prime. Circle them.

If you study the pattern of new 'mark-outs,' you'll see that you can predict what your first mark-out will be for any number. If this first mark-out is larger than the largest number on your page, you have finished the page, and any remaining unmarked numbers will be prime! By studying the patterns you will realize that in factoring a number, if you have begun with 2 and worked up to the square root of the number without finding a factor, then the number is prime!