

## Pythagorean Theorem

The Pythagorean Formula is more than  $a^2 + b^2 = c^2$ . Basically, it serves four purposes.

- 1.) If you know you have a right triangle, and you know the lengths of two sides, the formula will enable you to find the third side. You will need to know which two sides you have in order to put them in the right place in the formula.

- a.) If the two legs are 7 and 24, then:

$$\begin{aligned}7^2 + 24^2 &= c^2 \\ \text{So } 49 + 576 &= c^2, \\ \text{and } 625 &= c^2. \text{ We take the square root of both sides, and } c = 25.\end{aligned}$$

- b.) If the hypotenuse is 26 and one leg is 10, then:

$$\begin{aligned}10^2 + b^2 &= 26^2 \\ \text{So } 100 + b^2 &= 676, \\ \text{and } b^2 &= 576. \text{ We take square roots and } b = 24.\end{aligned}$$

It doesn't matter whether you put the 10 in for a or b, but the hypotenuse always replaces c in the formula.

- 2.) If you know all three sides of any triangle, if the Pythagorean formula is true, you know it is a right triangle. Let's say you have three sides of 8, 15, and 17.

$$\begin{aligned}a^2 + b^2 &\neq c^2 \\ 8^2 + 15^2 &\neq 17^2 \\ 64 + 225 &\neq 289 \\ 289 &= 289 \quad \text{Since it is equal, we have a } \mathbf{right\ triangle!}\end{aligned}$$

- 3.) If you know all three sides of any triangle, and the Pythagorean formula is false, you know it is an acute triangle if  $a^2 + b^2 > c^2$ . If we think of sides a and b as being "the little guys," and remember that acute angles are also "little," we can consider the formula as a contest between two "little guys" and one "big guy." Let's look at a triangle with sides of 8, 9, and 11.

$$\begin{aligned}a^2 + b^2 &\neq c^2 \\ 8^2 + 9^2 &\neq 11^2 \quad (\text{Notice that we put the sides in order}) \\ 64 + 81 &\neq 121 \\ 145 &> 121 \quad \text{The "little guys" won, so we have an } \mathbf{acute\ triangle!}\end{aligned}$$

- 4.) If you know all three sides of any triangle, and the Pythagorean formula is false, you know it is an obtuse triangle if  $a^2 + b^2 < c^2$ . Like the last example, a and b are the "little guys" and c is the "big guy." If the three sides of a triangle are 7, 10, and 15, let the contest begin!

$$\begin{aligned}a^2 + b^2 &\neq c^2 \\ 7^2 + 10^2 &\neq 15^2 \quad (\text{Again, notice that we put the sides in order}) \\ 49 + 100 &\neq 225 \\ 149 &< 225 \quad \text{Since the "big guy" won, we have an } \mathbf{obtuse\ triangle!}\end{aligned}$$

One other note: Any time you have three numbers, in order for them to form a triangle, the two smallest numbers must add up to more than the largest number.

See our "Pythagorean Seesaw" for more on this.