Pythagorean Triples (Primitive)

A primitive Pythagorean triple is a set of three whole numbers that make the Pythagorean Theorem true. Also, the three numbers will not have a common factor other than 1. Every "Primitive" triple will always consist of two odd numbers and one even number.

Examples: (Where "c" is the hypotenuse.)

Primitive (Table 1):

a	b	c
3	4	5
5	12	13
7	24	25
9	40	41
. . . (etc.)		

Mixed (Table 2):

a	b	c	
6	8	10	
8	15	17	(primitive)
10	24	26	
12	35	37	(primitive)
14	.	.	

Pythagorean triples exhibit many patterns.
In the first table, the "a" values are odd numbers; the "b" values increase in a pattern based on four, and the "c" values are one more than the b values.

In the second table, the "a" values are even, and every other triple is formed by multiplying table 1 by two. The triples that are not formed from table 1 are primitive triples (no common factors). Notice that the "b" values increase by odd numbers (7, 9, 11…) and the "c" values are now two more than the b values.

Looking at table 1, we can see that if we square "a," the answer is the same as b + c. Since c is one more than b, we can replace c with "b + 1." Therefore, we can set up the formula: $a^2 = b + b + 1$, or $a^2 = 2b + 1$. Or if we solve for b, we get $b = \frac{a^2 - 1}{2}$

Now, if we choose any odd number for "a," such as 25, then we square it (625), subtract 1 (624), and divide by two to find b (312), and c is one more (313). Therefore, 25, 312, and 313 should satisfy the Pythagorean Theorem:

$25^2 + 312^2 = 313^2$
$625 + 97344 = 97969$ and so it does.

We can develop a formula for any table by observing the patterns formed by each column and row. The formulas will be similar, but will change as each table changes.

Note: The two tables shown here are NOT the only tables of Pythagorean triples.

Table 1:
$b = \frac{a^2 - 1}{2}$

Table 2:
$b = \frac{a^2 - 2}{4}$

We can generalize the formula to say that if we square our shortest side (a), then divide that by <u>twice the difference</u> of b and c, then subtract one half, we will have the length of the next side (b). We then add the common difference to b to find the hypotenuse.