Simplifying Radicals

Most students know the mnemonic for order of operations as "Please excuse my dear Aunt Sally." We know that Addition and Subtraction are opposites, as are Multiplication and Division. We also know that the "e" in PEMDAS is "exponent," but where is its opposite?

If we write our PEMDAS in the fashion at right, we emphasize that addition and subtraction are on the same level; multiplication and division are also, but on a higher level. Exponents and their opposites (roots or radicals) are on an even higher level.*

P
E(R)
MD
AS

Opposite operations eliminate each other. Everyone knows $\frac{5x}{5}$ = x and x + 7 – 7 = x.

Most don't realize that exponents and roots work the same way: $\sqrt{x^2} = x$ and $(\sqrt{x})^2 = x$.
What this means is that if you want to know the square root of 17^2, you <u>don't</u> have to know that 17^2 is 289 and the square root of 289 is 17. Skip the middle steps and let the exponents and roots eliminate each other (or as some say "cancel out").

But what if the number in the radical is not a perfect square, or maybe not one that you know? Then we must break it down.

Step one: Are there any obvious perfect squares as factors?

$\sqrt{3600}$ can be seen as $\sqrt{36 \cdot 100}$, or $\sqrt{36} \cdot \sqrt{100}$, which is 6 ·10, or 60.

$\sqrt{75}$ is $\sqrt{25 \cdot 3}$, or $\sqrt{25} \cdot \sqrt{3}$, which is $5\sqrt{3}$, which won't simplify further.

Step two: If perfect squares are not obvious, break it down into prime factors.* By definition, any factor which is used twice is **<u>squared</u>**, so we can take its square root.

$\sqrt{180}$ can be broken down to $\sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}$, or $\sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{5}$, or $2 \cdot 3 \cdot \sqrt{5} = 6\sqrt{5}$.

$\sqrt{56} = \sqrt{2 \cdot 2 \cdot 2 \cdot 7}$. Since we're doing square root, we group by pairs. $\sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{7}$, or $2\sqrt{14}$.

Do you remember how in the board game Monopoly, you got a "Get Out of Jail Free" card? I like to think that the exponent can serve a similar purpose. When you're doing square root, every factor that has an exponent of 2 holds a "Get Out of Radical Free!" This notion helps us when we move into algebraic symbols.

$\sqrt{4x^2y^3z^4} = \sqrt{2^2 \cdot x^2 \cdot y^2 \cdot y \cdot z^2 \cdot z^2}$. Notice that everything has a "GORF" except for one poor little y. Therefore everything comes out except that y, and the z comes out twice! Our answer is: $2xyz^2\sqrt{y}$

This method works no matter what root you are taking. For example, if you are taking cube root, you group the factors by threes. So our last example has a totally different answer if we do cube root. The factors now get a "GORF" with an exponent of 3.

$\sqrt[3]{4x^2y^3z^4} = \sqrt[3]{4x^2y^3z^3z}$, so our answer is $yz\sqrt[3]{4x^2z}$

Everyone should know:

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

$11^2 = 121$

$12^2 = 144$

$13^2 = 169$

$14^2 = 196$

$15^2 = 225$

* See our study guide on this subject.