

Subtraction

(“Borrowing,” Bumping, and Adding)

Most students are taught to subtract large numbers by a process which was once called “borrowing.” The politically correct term is now, I think, “Renaming.” I prefer to think of it as making change. If the instructor will print off enough bills of correct denominations, this can be taught using manipulatives. Suppose we want to subtract 39,865 from 47,603.

$\begin{array}{r} 57603 \\ -39865 \\ \hline \end{array}$	becomes	$\begin{array}{r} 93 \\ 57\cancel{6}0\cancel{3} \\ -39865 \\ \hline \end{array}$	Which becomes... which we now subtract.	$\begin{array}{r} 165 \\ 5\cancel{6}13 \\ 57\cancel{6}0\cancel{3} \\ -39865 \\ \hline 17738 \end{array}$	Picture this as having five \$10,000 bills, seven \$1000 bills, six hundreds, no tens and 3 ones.
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If you owed someone \$39,865 and met them in a bank, you could pay them the exact amount by getting change for one hundred, one \$1000, and one \$10,000. Then you have to get change for one of your tens. They will get three \$10,000 bills, nine \$1000s, eight hundreds, six tens, and five ones. You’ll get to keep \$17,738. (You wish!)

Method two is called the “Austrian Method.” The name alone makes it worth teaching. Instead of “borrowing” from the top row, we “bump” the bottom row up by one. Think of this as having two numbers that are a fixed amount apart. If we increase one number by ten, then we must increase the other by ten to maintain the difference. Using the same numbers as above, we can increase the top number by adding ten ones. If we add a ten to the bottom number, the difference remains the same. It looks like this:

$\begin{array}{r} 57603 \\ -39865 \\ \hline \end{array}$	becomes	$\begin{array}{r} 93 \\ 5760\cancel{3} \\ -398\cancel{6}5 \\ \hline 8 \end{array}$	$\begin{array}{r} 93 \\ 5760\cancel{3} \\ -398\cancel{6}5 \\ \hline 38 \end{array}$	$\begin{array}{r} 165 \\ 5\cancel{6}13 \\ 57\cancel{6}0\cancel{3} \\ -39865 \\ \hline 17738 \end{array}$
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As I work from right to left, I say, “Thirteen minus 5 is 8, ten minus 7 is 3, sixteen minus 9 is 7, seven minus zero is 7, and five minus 4 is 1.” The advantage to this is that addition is easier than subtraction. I still have to subtract to do the problem, but addition gives me the numbers to subtract. It takes practice, but is faster and better once mastered.

Method three has no name that I know of. Its advantage is that it requires almost no subtraction, and no “borrowing” at all. Using the same numbers as above...

$\begin{array}{r} 57603 \\ -39865 \\ \hline \end{array}$	We can subtract 5-3, but the rest cause problems so we rewrite it as this...	$\begin{array}{r} 50000 + 7603 \\ -39865 \\ \hline \end{array}$	Now take 1 from the 50000 and add it to the number on the right. The total is unchanged.	$\begin{array}{r} 49999 + 7604 \\ -39865 \\ \hline 10134 \\ + 7604 \\ \hline 17738 \end{array}$
We then add the number on the right back onto this answer. →				←
Same answer as before!!				