

## Systems of Equations

A system of equations is usually\* two linear equations which are solved simultaneously to find out if the lines cross, and if so, where they cross. The solution is an ordered pair that indicates where the two lines intersect.

We use three methods to solve the systems:

**Graphing** – We graph both lines and see where they cross. If the ordered pair is not integers, this method is not very useful. (See example 1.)

**Substitution** – We solve at least one of the equations for one variable, then substitute that expression into the other equation. This can get plumb hairy! (Example 2.)

**Subtraction** – We manipulate one or both equations so the coefficients of one variable are additive inverses (opposites). When we add the equations together, that variable subtracts out, leaving us one equation and one variable. (Example 3.)

### Example 1

System:

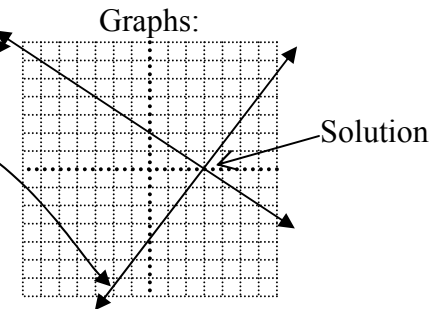
$$2x + 3y = 6$$

$$4x - 3y = 12$$

(On equation 1, the slope is  $-\frac{2}{3}$  and the y-intercept is 2.)

(On equation 2, the slope is  $\frac{4}{3}$  and the y-intercept is -4.)

Close examination shows the lines cross at (3, 0).



### Example 2

System:

$$2x + 3y = 6 \quad \text{Solve for } x: \quad x = -\frac{3}{2}y + 3 \quad \text{or solve for } y: \quad y = -\frac{2}{3}x + 2$$

$$4x - 3y = 12 \quad \text{Substitute:} \quad 4\left(-\frac{3}{2}y + 3\right) - 3y = 12 \quad \text{or} \quad 4x - 3\left(-\frac{2}{3}x + 2\right) = 12$$

$$\text{Solve:} \quad -6y + 12 - 3y = 12 \quad \quad \quad 4x + 2x - 6 = 12$$

$$\quad \quad \quad -9y + 12 = 12 \quad \quad \quad 6x - 6 = 12$$

$$\quad \quad \quad -9y = 0 \quad \quad \quad 6x = 18$$

$$\quad \quad \quad y = 0 \quad \quad \quad x = 3$$

Solution: (3, 0)

### Example 3

System:

$$2x + 3y = 6$$

$$4x - 3y = 12$$

$$6x = 18$$

$$\text{so... } x = 3$$

Note that the coefficients of y are opposites, so all we have to do is add the equations and the y values make zero. Sometimes we need to multiply one or both equations to make this happen.

We then substitute 3 in place of x in either equation.

$$\text{Equation 1: } 2(3) + 3y = 6 \quad \text{or} \quad \text{Equation 2: } 4(3) - 3y = 12$$

$$6 + 3y = 6 \quad \quad \quad 12 - 3y = 12$$

$$3y = 0 \quad \quad \quad -3y = 0$$

$$y = 0 \quad \quad \quad y = 0$$

Solution: (3, 0)

\* Any two (or more) equations can form a system, such as a parabola and a line. Your solution(s) is where they cross.